

# RISE

RESEARCH ON IMPROVING  
SYSTEMS OF EDUCATION

## Simulating learning: A formal model for learning profiles, with applications for understanding teacher value added

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Research on Improving Systems of Education (RISE) Programme

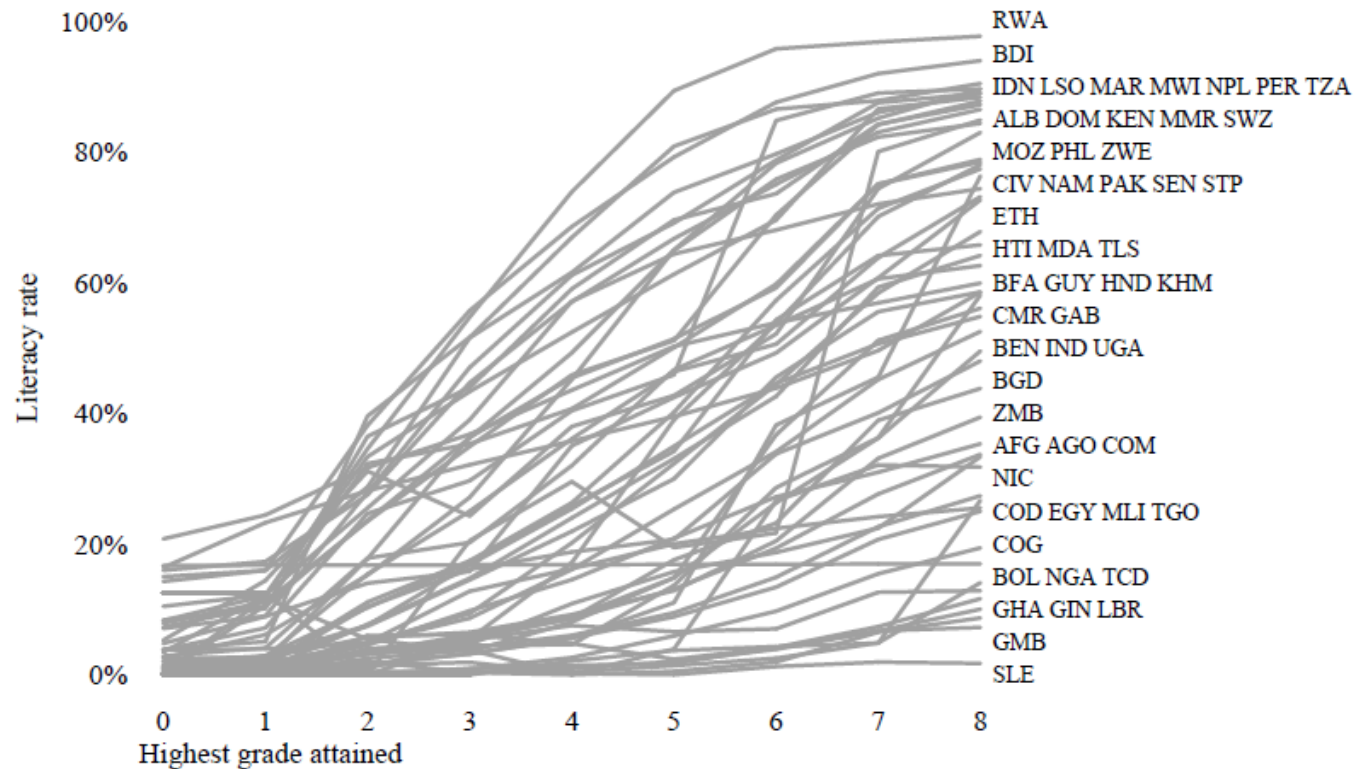
Lant Pritchett

University of Oxford



# Learning profiles show that learning is highly varied across countries and on average is low

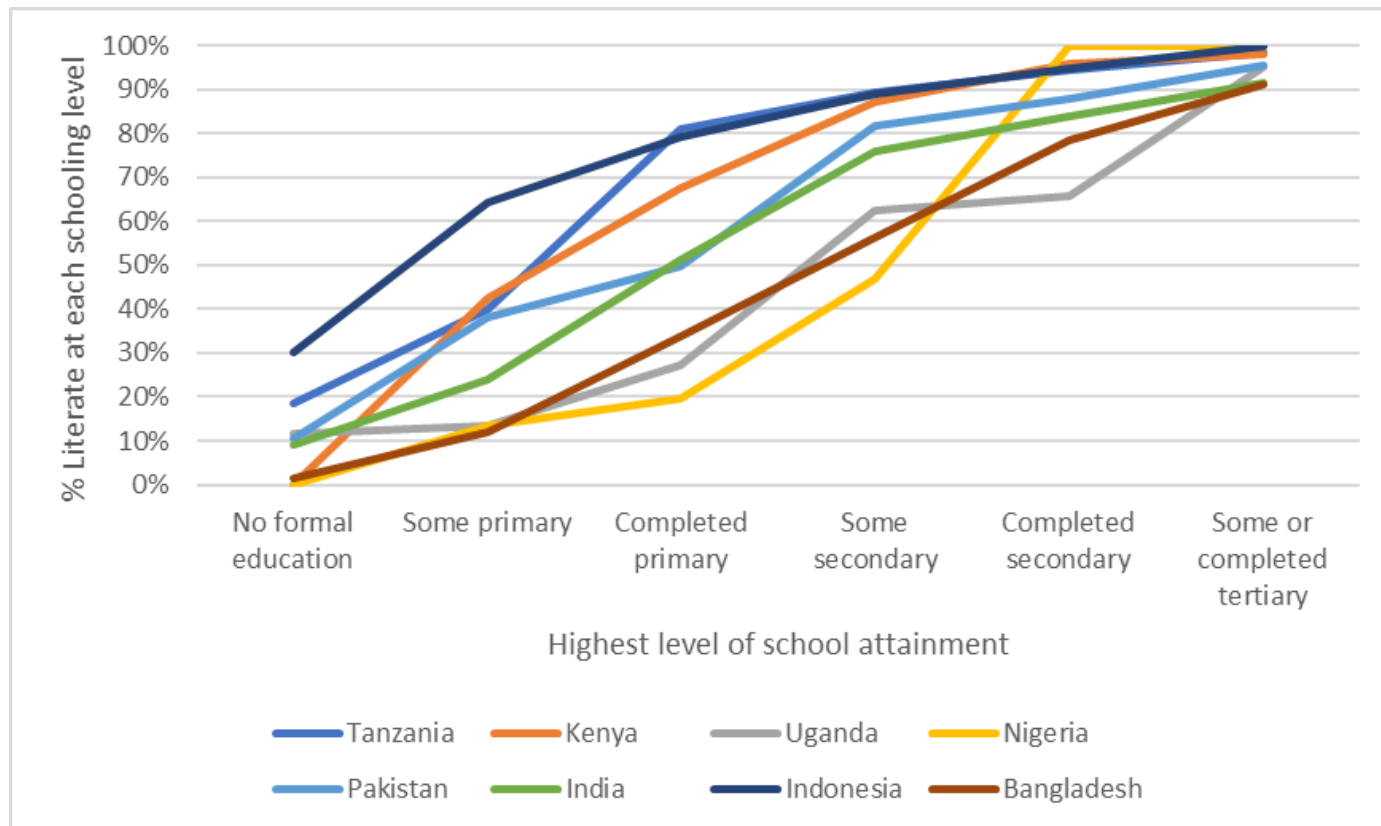
Figure 1: Learning profiles from DHS data show literacy among young women with six years of schooling varies from less than 10% to nearly 100%



Source: Pritchett and Sandefur 2017

# Findings are consistent: Similar findings from different data using different literacy assessment

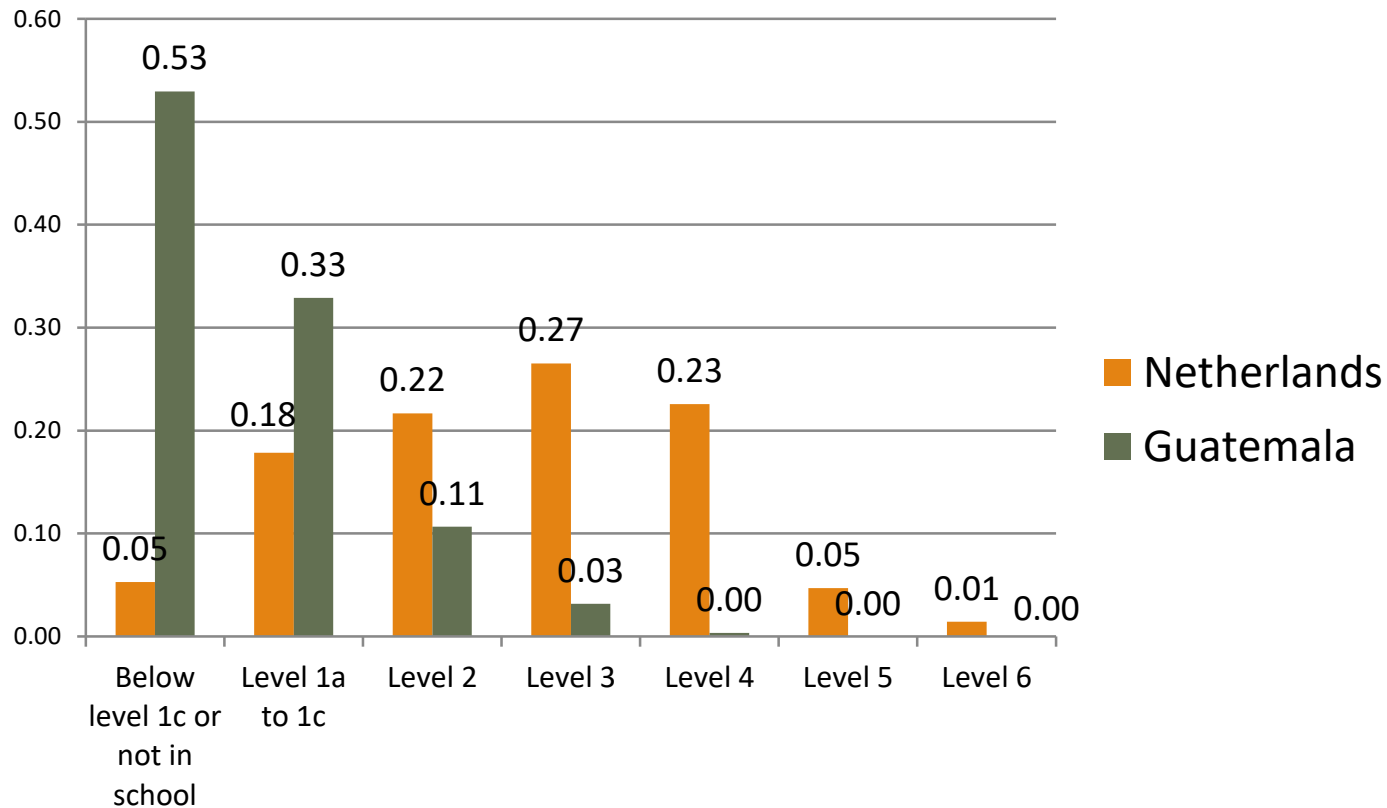
Figure 2: Literacy among adults with primary completion as their highest attainment varies from 20% to 80%



Source: Kaffenberger and Pritchett 2017, using Financial Inclusion Insights data

# Recent PISA-D data show developing countries are far behind any “universal basic mastery” goal

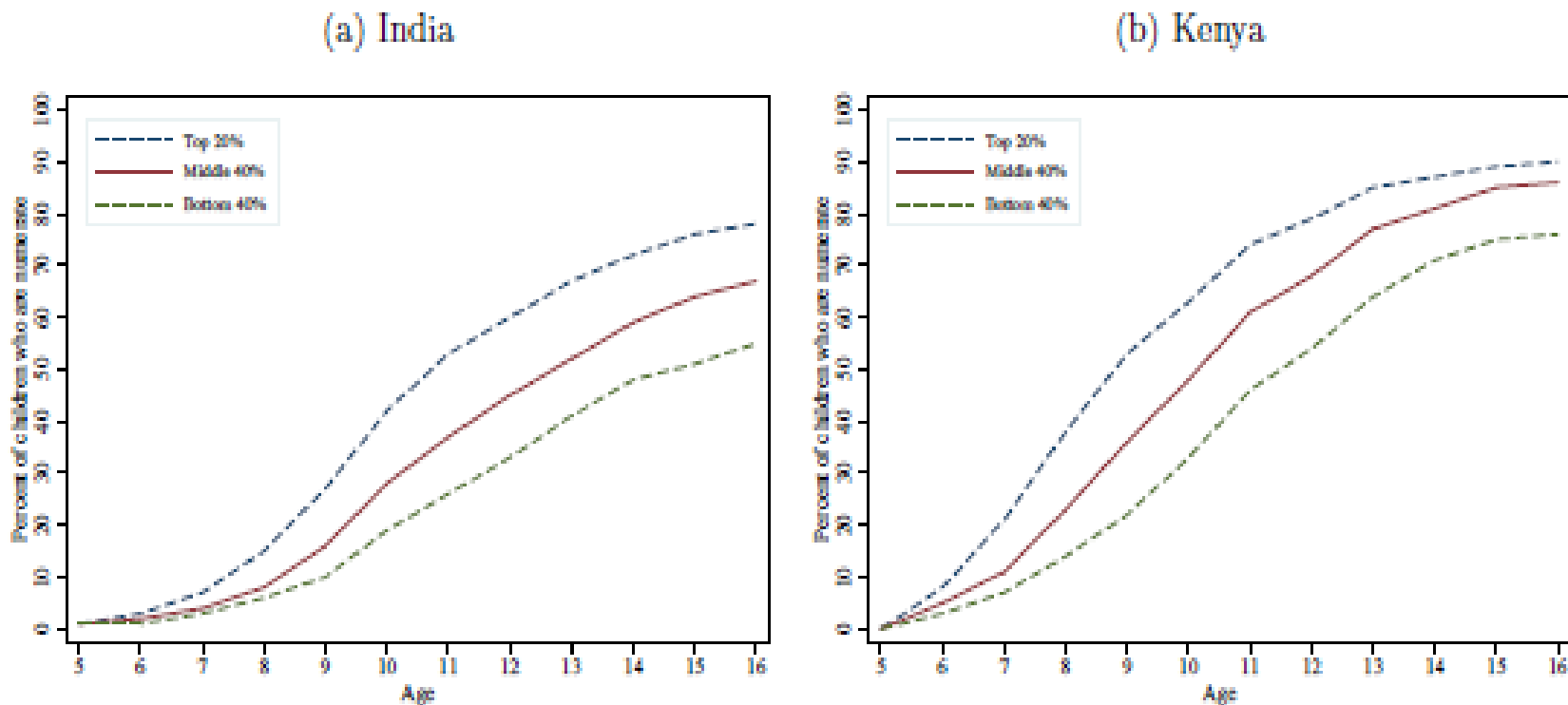
Figure 3: 97% of 15-year-olds in Guatemala are “low skill” by OECD standards, and essentially none are in the top three levels



Source: Pritchett 2019

# Learning also varies substantially within countries

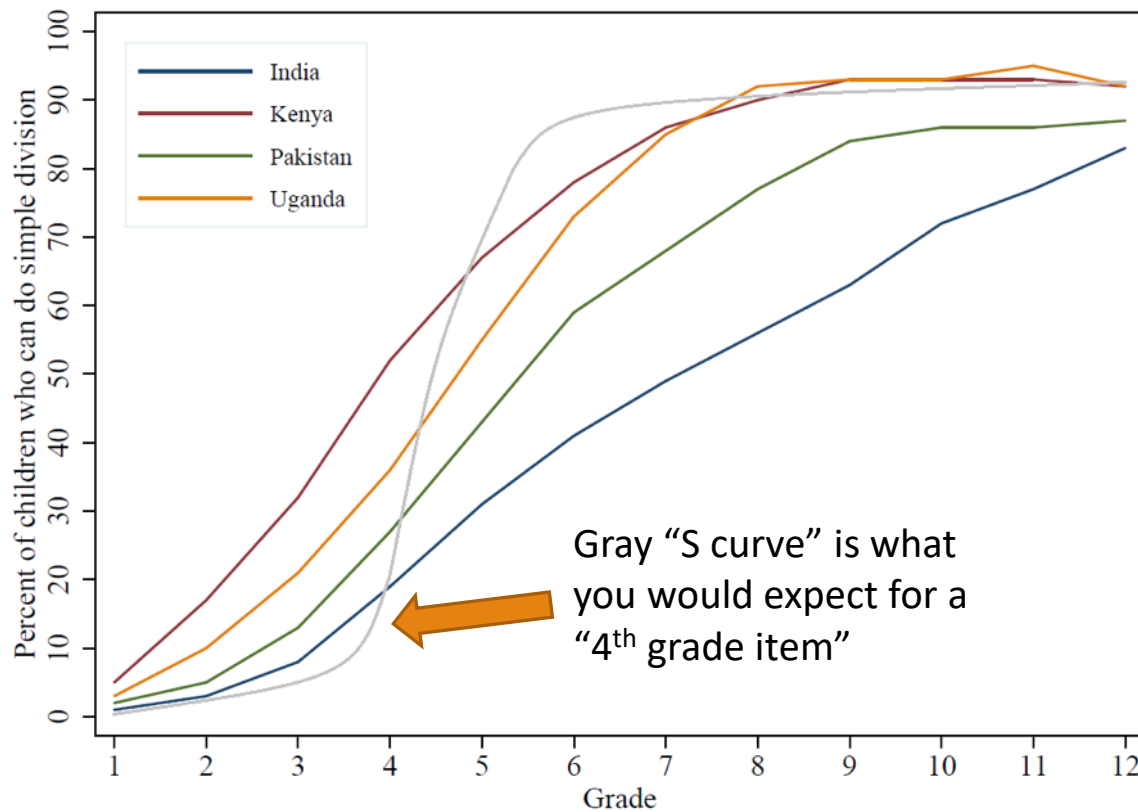
Figure 4: Learning trajectories by age and household wealth: Math



Source: Akmal and Pritchett 2019, using ASER and Uwezo data

# Learning trajectories often take a different “shape” than expected: Learning a specific item is spread over many grades

Figure 5: Why are 8<sup>th</sup> graders in India and Pakistan still learning a 4<sup>th</sup> grade item?



For a “4<sup>th</sup> grade item” we would expect small incline a grade or two before (high performers), large increase in fourth grade, and small increase for a grade or two after (remedial learners). Instead we often see linearity.

Source: Akmal and Pritchett 2019

# What explains these learning trajectories?

- Learning outcomes are driven by:

1) the initial distribution of student skills, and

2) an instructional process that imparts some level of learning for a child at each point in that distribution

# What explains these learning trajectories?

In this paper we:

- Develop a formal model that characterizes this instructional process.
- Use the model to simulate learning profiles; replicate observed learning profiles
- Show implications of the model for understanding teacher value added – showing that multiple factors, not just teacher ability, determine observed TVA, with implications for improving learning.



# The Potential Pedagogical Function

Building on Beatty and Pritchett (2012) we construct a potential pedagogical function (PPF)

- Models the amount a child at each point in the student distribution learns during an increment of schooling
- Characterized by four main elements:
  - **Height:** Maximum that can be learned during an increment of schooling
  - **Shape:** Determines how much children at different points in the distribution learn
  - **Range:** Range of student abilities that learn under the PPF (children outside the range learn nothing; e.g. a child who can't recognize numbers won't learn from instruction on division)
  - **Location/centeredness:** Level of student skills the PPF is targeted for or centered on

# Potential Pedagogical Function Variation 1

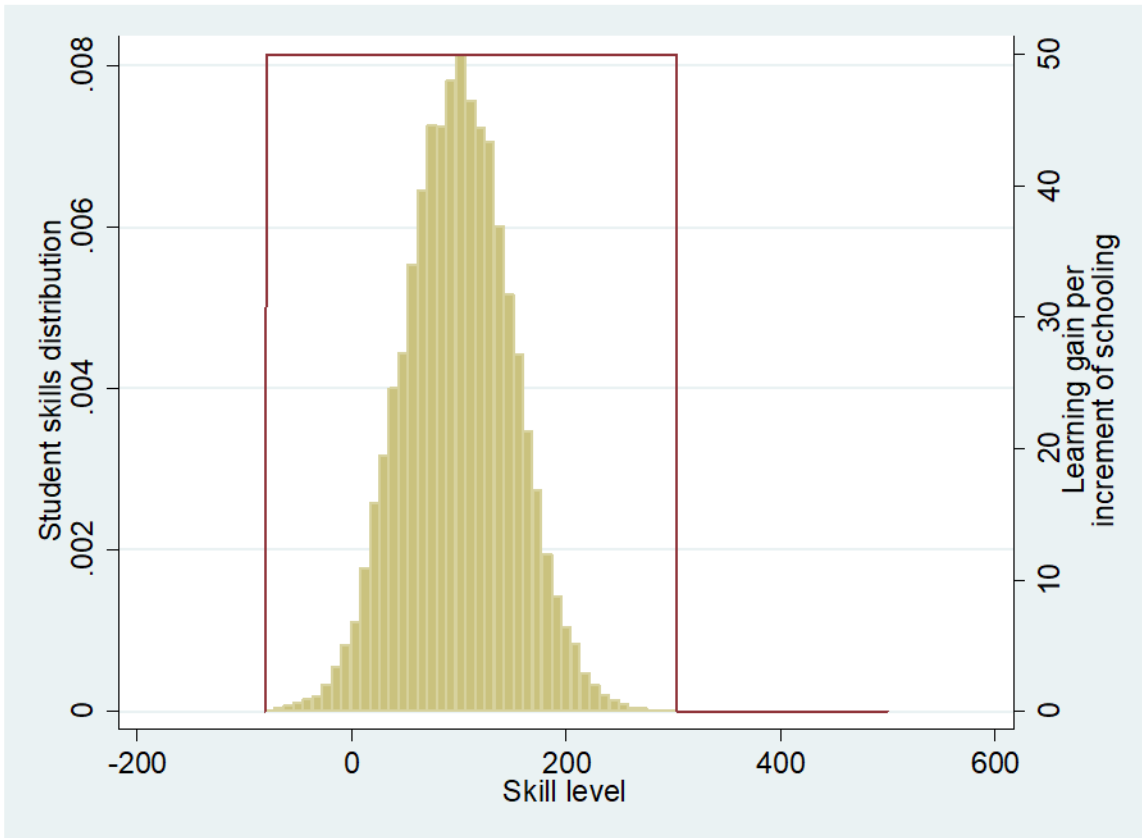
PPF<sub>1</sub>: Simplest PPF, Rectangular shape, all children within the PPF range learn the same

The learning of any student “i” of initial skill “s” is expressed as a piecewise linear equation that is a function of the PPF’s maximum height ( $h_{max}$ ) and its range which we define by its endpoints a and b:

$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{max} & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

# PPF<sub>1</sub>: Rectangular PPF, encompassing the full student distribution

Figure 6: Simplest PPF where all children learn the same



Source: Authors' simulations

$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{max} & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

Parameters constructed to encompass full student distribution:

$$\begin{aligned} a &= student_{min} \\ b &= student_{max} \\ h_{max} &= 50 \end{aligned}$$

Illustrated over a student distribution of mean 100 stdev 50

# PPF Variation 2: Able to replicate OECD PISA scores

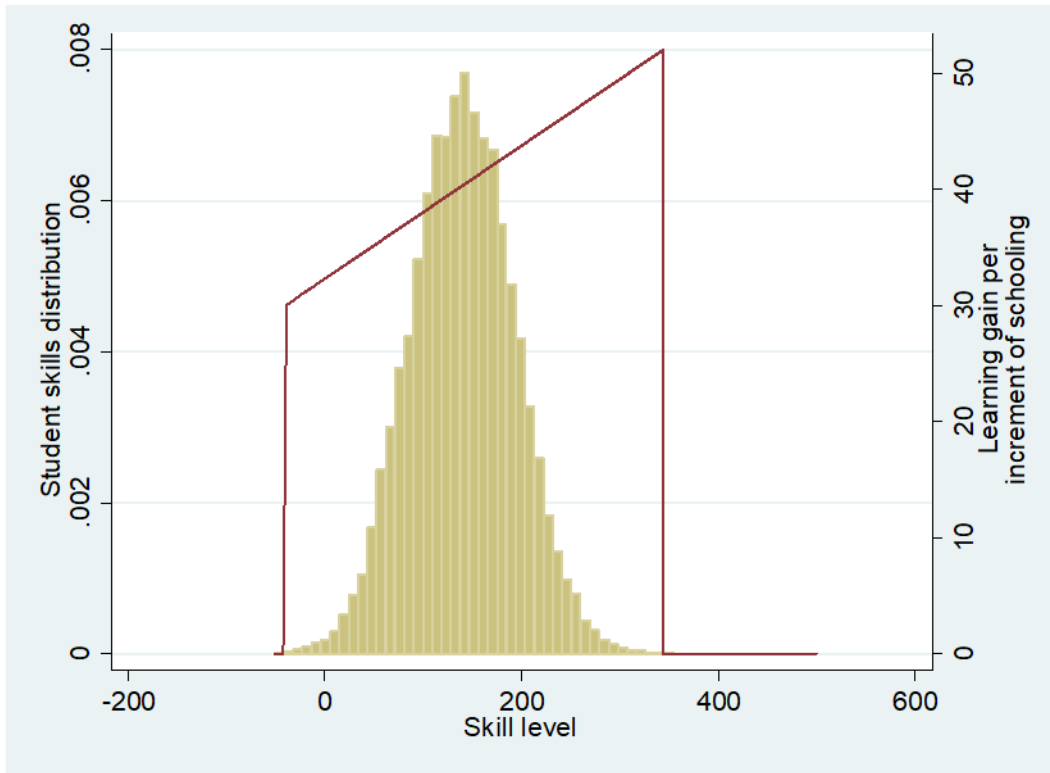
A PPF in which all children within PPF range learn a minimum; learning increases linearly with student initial ability.

Represented by expression:

$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{min} + (h_{max} * (s^i - a)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

# PPF<sub>2</sub>: All children learn a minimum amount, but high performers learn more

Figure7: Instruction increases linearly with initial ability



$$L = \begin{cases} 0 & \text{if } s^i < a \\ h_{min} + (h_{max} * (s^i - a)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

Parameters constructed to encompass full student distribution:

$$a = student_{min}$$

$$b = student_{max}$$

$$h_{max} = 52$$

$$h_{min} = 30$$

Illustrated over a student distribution of mean 100 stdev 50

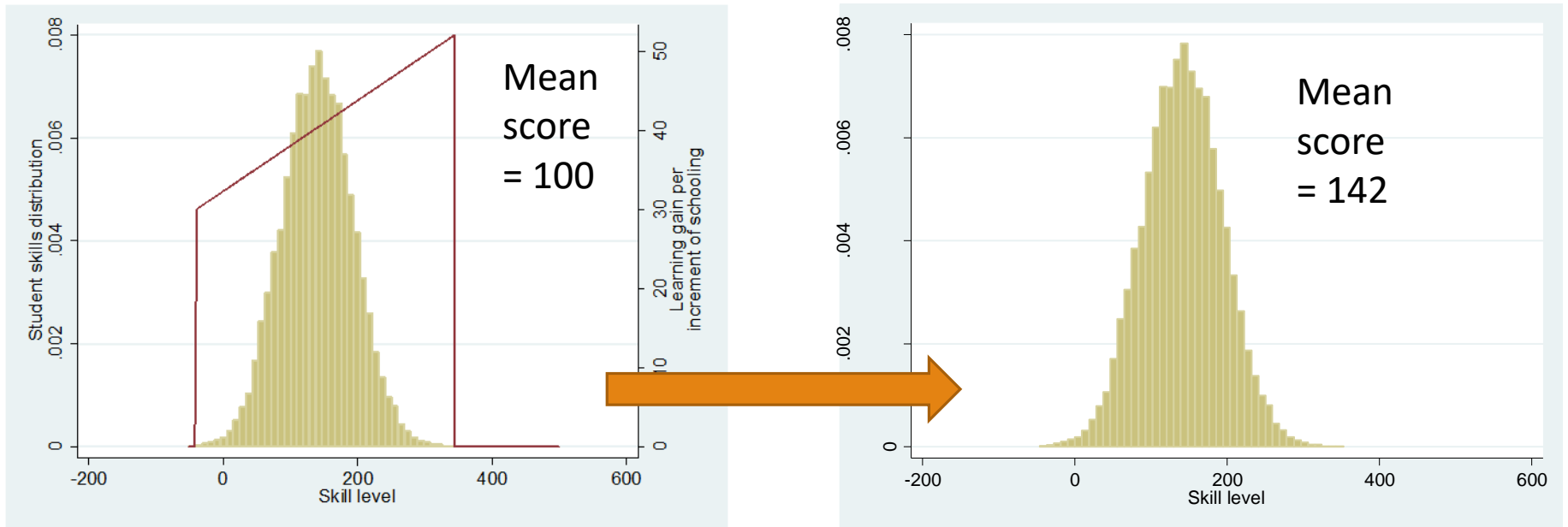
Source: Authors' simulations

# Simulating learning outcomes

- Schooling is a series of instructional processes.
- We use our model to simulate 12 years of schooling, applying the PPF to the initial student distribution to produce a new student distribution; shifting the PPF to represent instruction at the next grade level; and apply it again; and we iterate this 12 times.
- Produce an average learning profile and disaggregated learning profiles by initial student ability level.

# Simulating learning: how it works

Figure 8: Students enter grade 1 with mean score of 100; learn an amount dictated by PPF (maximum of 52, minimum of 30, with an average gain of 42 in below example), and then enter grade 2 with a mean score of 142



# Simulation parameters

Define four main parameters:

- Height ( $H_{\max}$ ,  $H_{\min}$ ), vary in following examples
- Range ( $\text{student}_{\min}$ ,  $\text{student}_{\max}$ )
- Location (centered)
- Shape (varies)

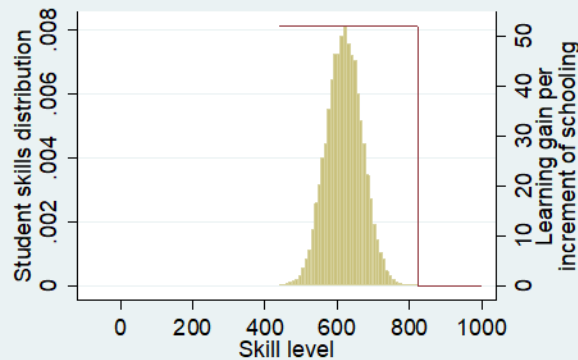
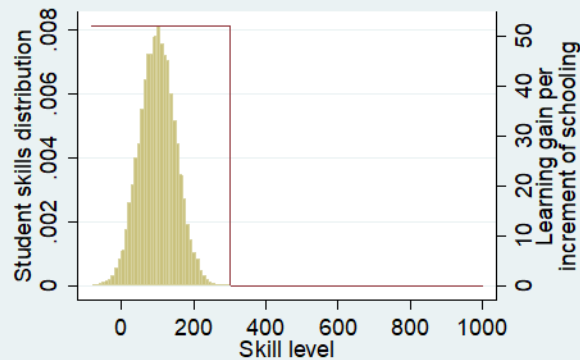
Additional parameters for simulations:

- **Pace** = amount PPF shifts for each “year” of schooling = median height, so median student keeps pace with instruction
- **Initial student distribution**: normal distribution with mean 100 standard deviation 50

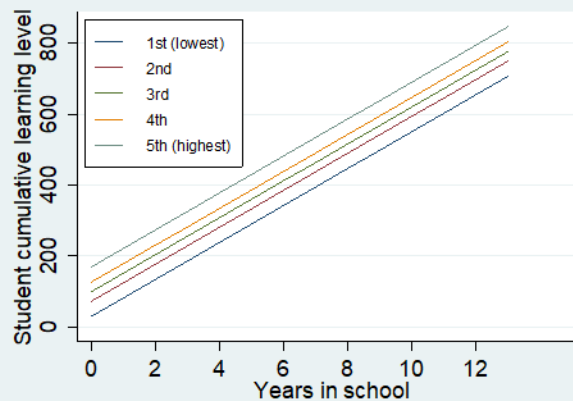
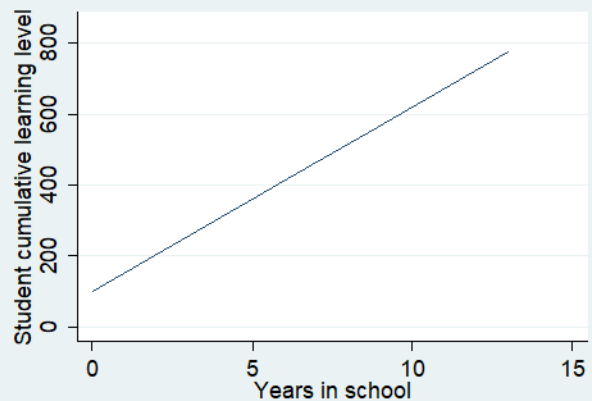


# PPF<sub>1</sub>: Rectangular, all children learn the same

Pedagogical function and student distribution at year zero and year 10



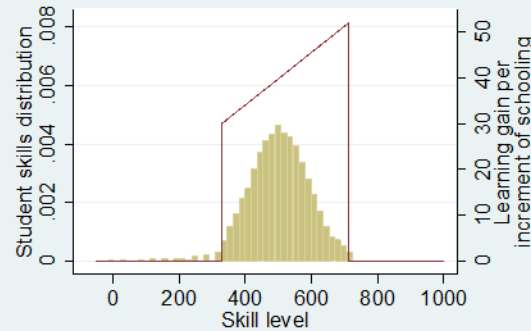
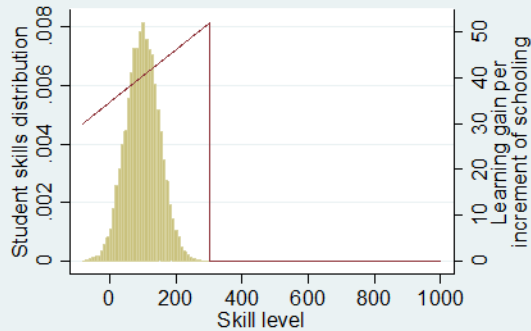
Learning profiles: Average, and by quintiles



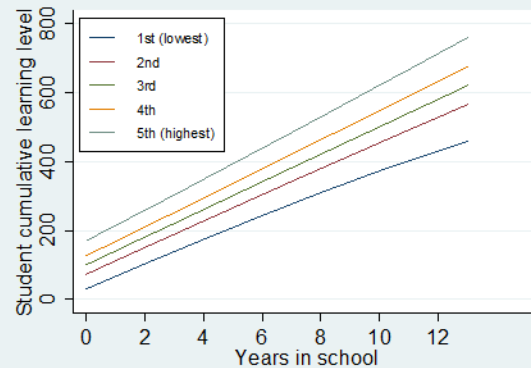
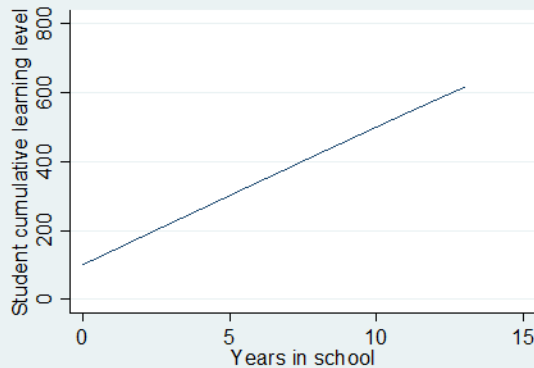
- Simplest PPF, all children learn the same amount in each year
- H=50, Pace=50
- Linear learning profiles
- Same slope for all learning profiles (by construction)

# PPF<sub>2</sub>: All children learn a minimum, but high performers learn more

Pedagogical function and student distribution at year zero and year 10



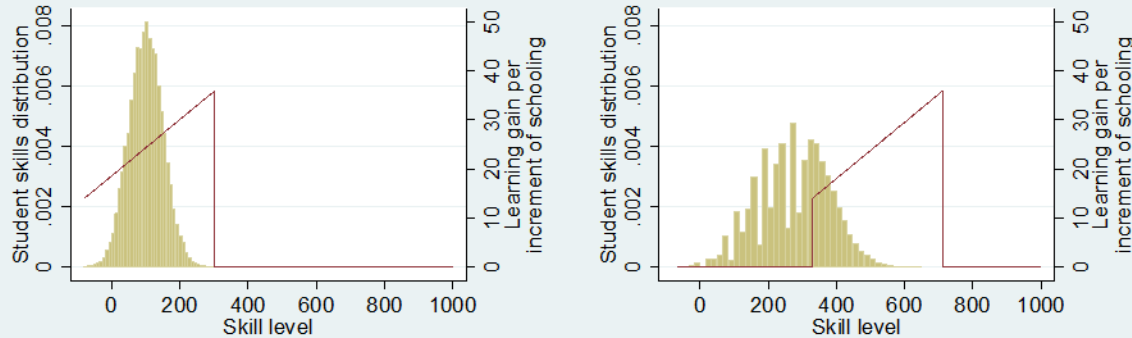
Learning profiles: Average, and by quintiles



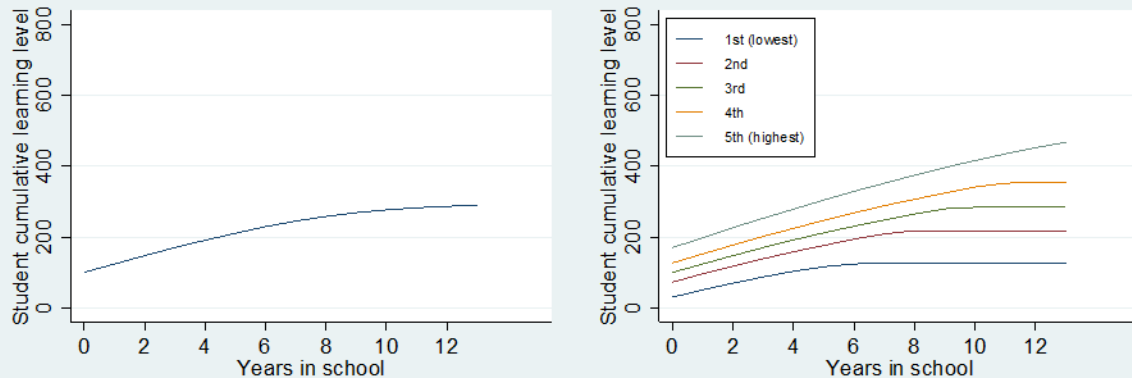
- Parameters calibrated to replicate OECD mean of 500 stdev 100 in grade 10
- By grade 10 some students are outside range and not learning
- Top quintile slightly steeper slope – learning more in each grade; bottom quintile slightly flatter slope – learning less in each grade

# Variation on PPF<sub>2</sub> : All children learn a minimum and high performers learn more, but at *Zambian* learning levels

Pedagogical function and student distribution at year zero and year 10



Learning profiles: Average, and by quintiles



- Parameters calibrated to replicate Zambia PISA-D reading score of 275 in grade 10 (lower PPF height, less learning at each point)
- By grade 10 majority of students are outside range and not learning
- Large variation in learning profiles by initial quintile, bottom quintile flattens by about 6<sup>th</sup> grade

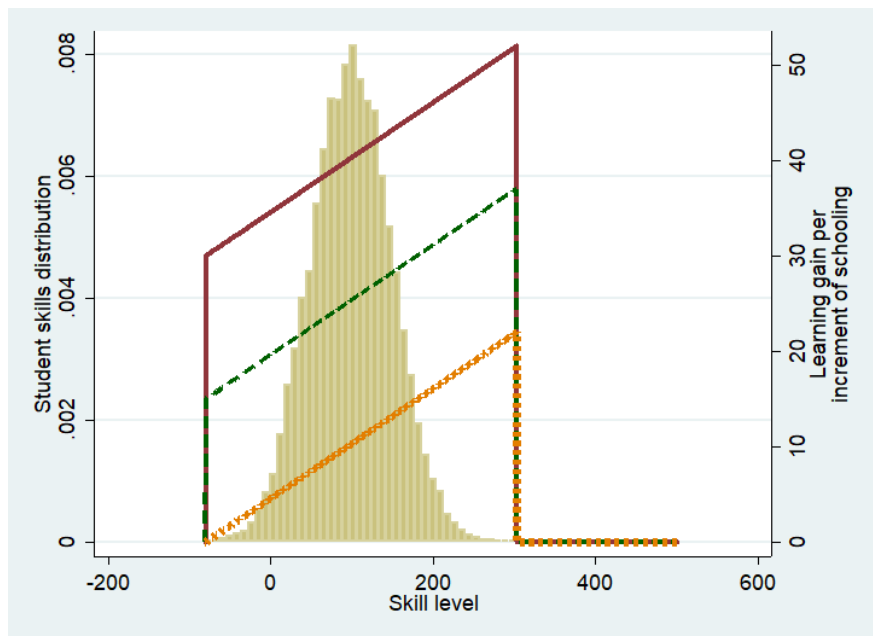
# Application to common understanding of teacher value-added

- PPF as a measure of TVA
- Every teacher has different level of ability – combination of innate ability, training, etc.
- PPF represents teacher’s ability to produce learning outcomes for students at each point in the distribution
  - Max height represents the most learning a teacher is able to produce
  - Range is the range of student abilities for which a teacher is able to produce learning
  - Area under PPF can be thought of as “total ability” to produce learning

# Applications to teacher value-added: Same shape and center, but varied height (learning production) at each point

- PPF as a measure of TVA
- “Typical” understanding of TVA: varying heights, or varying “teacher ability”, varying the total area under the PPF

3 PPFs of varying height; other parameters constant

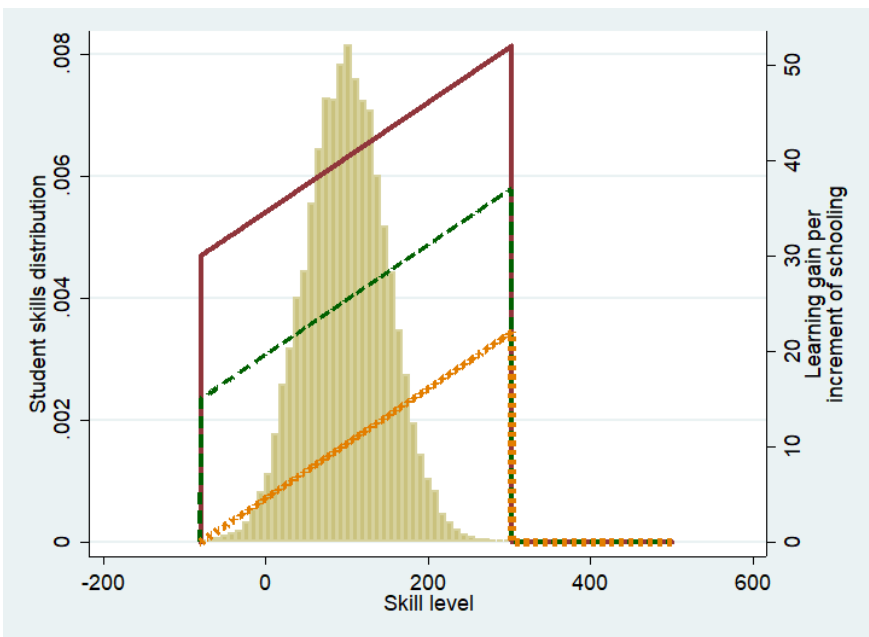


- Red calibrated to replicate OECD scores of mean 500 stdev 100 in grade 10
- Green and yellow PPFs produce lower learning for students at each point in distribution
  - Green = middle-performing teacher; yellow = low-performing teacher

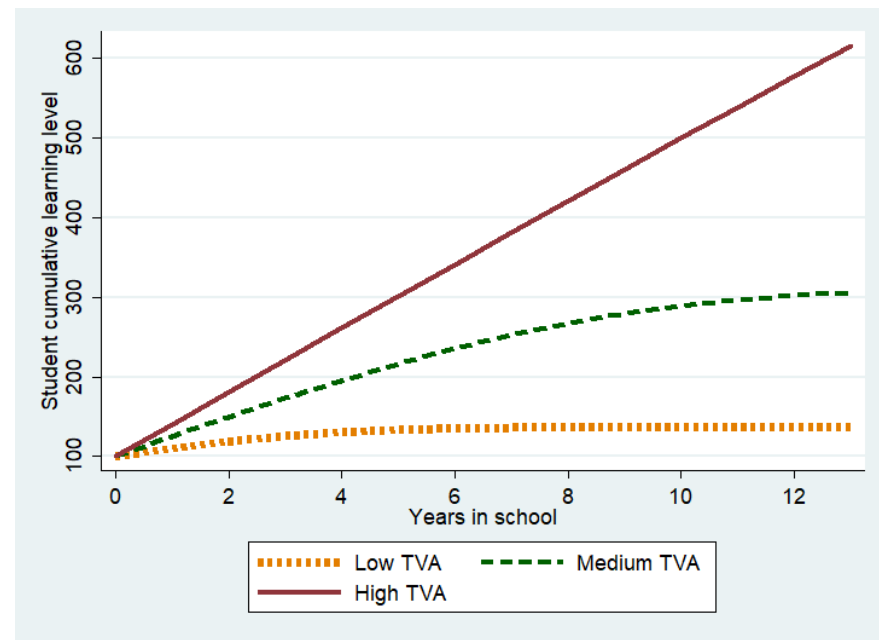
# Applications to teacher value-added: Same shape and center, but varied height (learning production) at each point

- Simulate PPFs across 12 years of schooling to produce learning profiles
- Red replicates OECD; green much lower, yellow extremely low
- If measuring TVA, observe learning levels for a given grade (not PPFs) and assume observing high- middle- and low-performing teachers

3 PPFs of varying height; other parameters constant



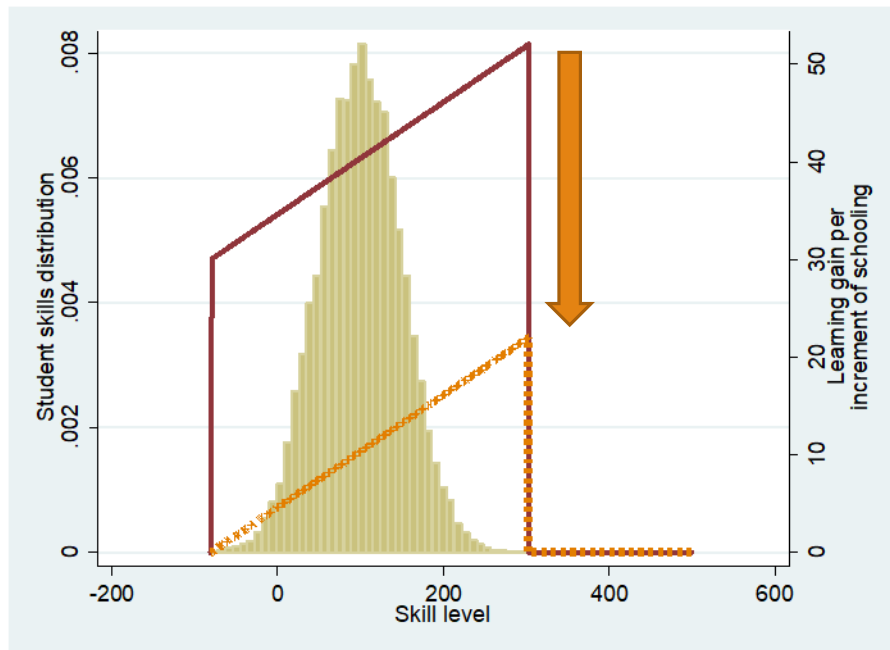
Observed TVA: Highest produces high learning, lowest produces very little learning



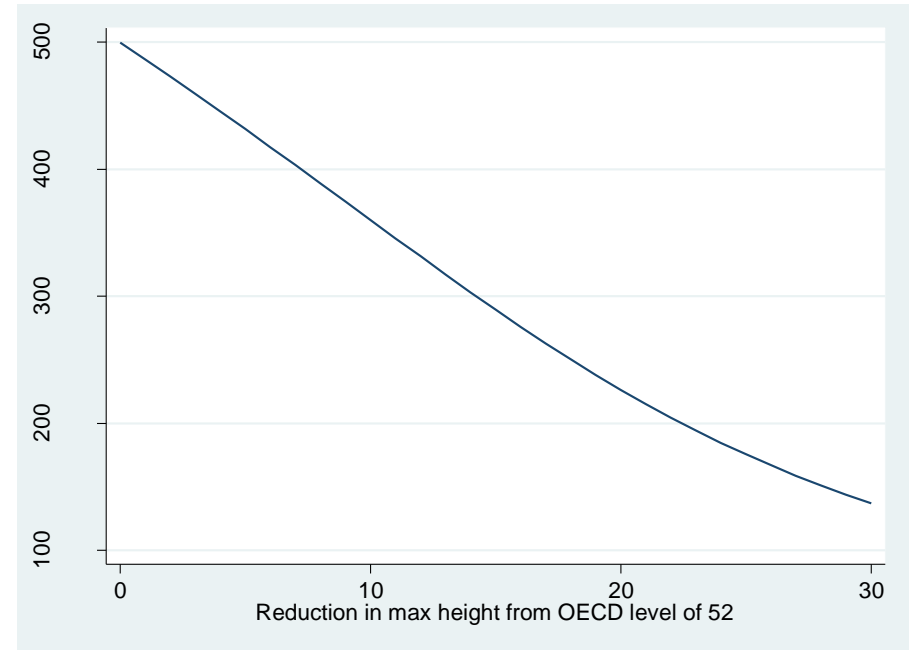
# What if we iterate through all heights within a given range, what average learning levels are produced?

- By varying only height (other parameters constant), can produce learning outcomes in grade 10 of everything from OECD scores to almost no learning at all

Vary PPF height from max height 52 to max height 22; other parameters constant



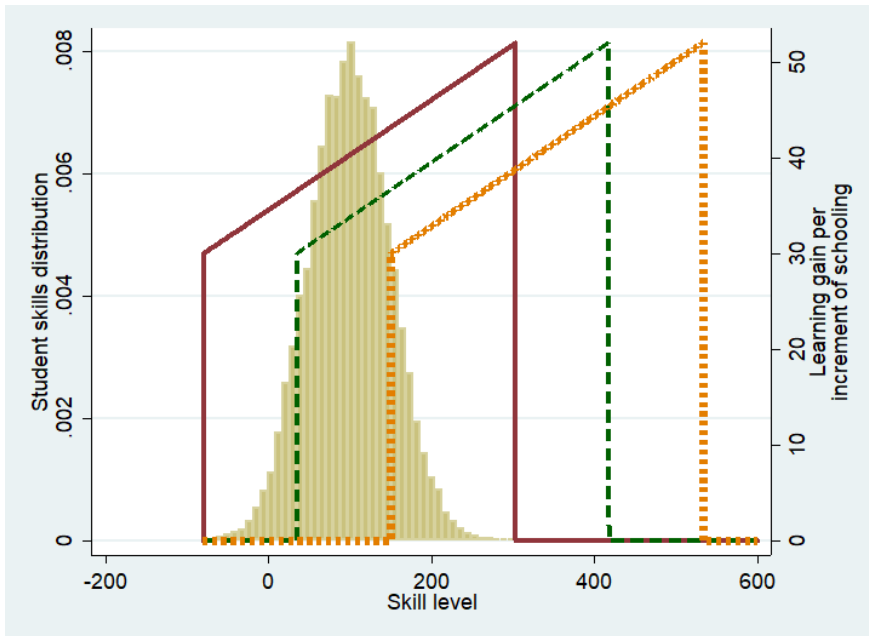
Starting from OECD average of 500, down to average cumulative learning of less than 150



# Now shape and size constant, but teaching not aligned with student ability

- 3 PPFs represent teachers of *equal abilities* – same height, same area
- Centeredness of instruction on the student distribution varies (with other parameters constant)
- Could represent overambitious curriculum, methods, etc.

3 PPFs with same area = same “ability”, but with varied centeredness



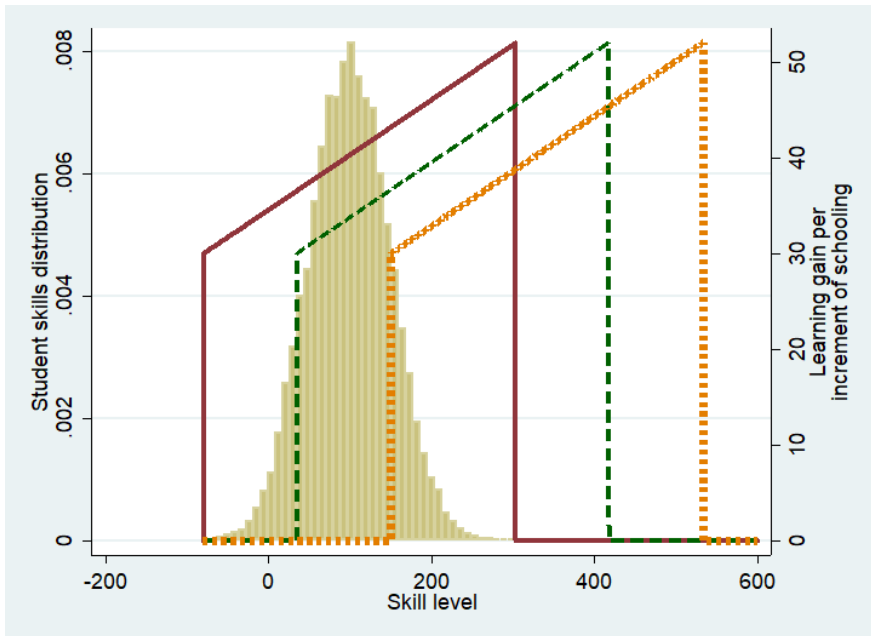
- Red calibrated to replicate OECD scores of mean 500 stdev 100 in grade 10
- Green = same *ability* (height, area) but off-center slightly; using curriculum or methods slightly ahead of student abilities
- Yellow = same *ability* (height, area) but off-center substantially; using curriculum or methods substantially ahead of student ability



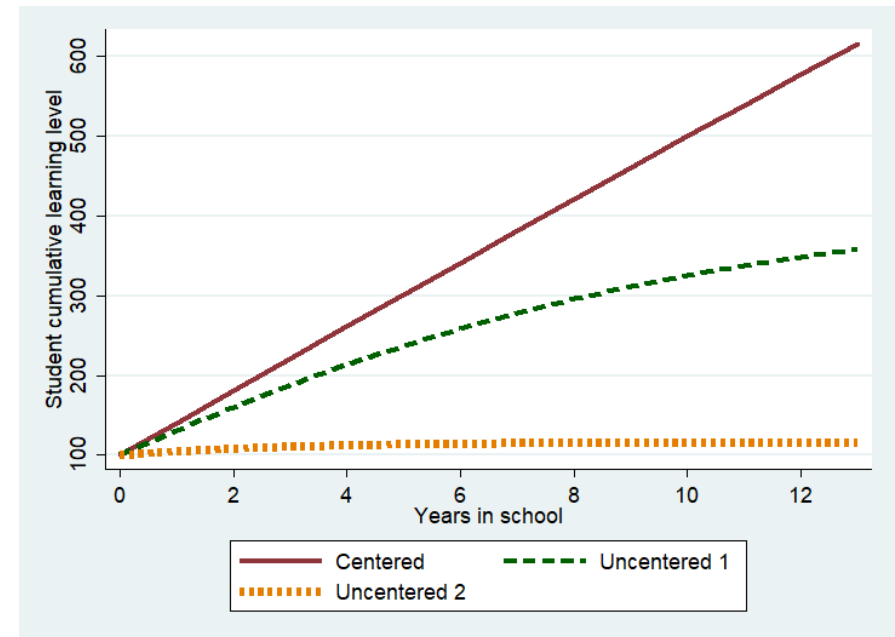
# Now shape and size constant, but teaching not aligned with student ability

- Simulate PPFs across 12 years of schooling to produce learning profiles
- Red replicates OECD; green and yellow have same “ability” as OECD teacher, but instruction isn’t centered, producing substantially less learning
- If measuring TVA, observe learning profiles for a given grade and assume observing high- middle- and low-ability teachers, when actually ability is constant and centeredness is the problem

3 PPFs with same area = same “ability”, but with varied centeredness



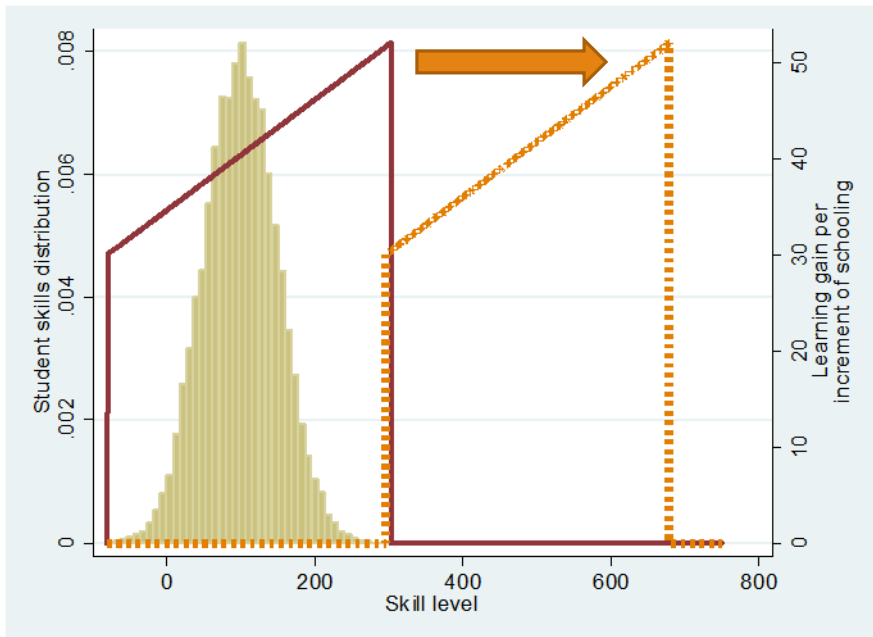
Observed TVA: Massive variation in learning outcomes for teachers of same potential ability



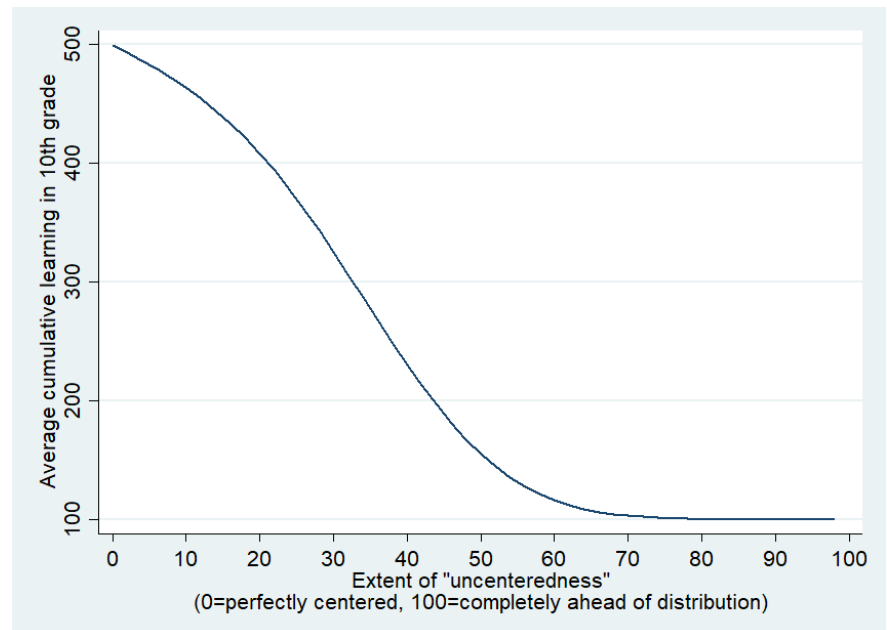
# What if we iterate across all locations within a given range, what average learning levels are produced?

- By varying only centeredness (other parameters constant), can produce learning outcomes in grade 10 of everything from OECD scores to almost no learning at all

Vary PPF centeredness from centered to fully uncentered; other parameters constant



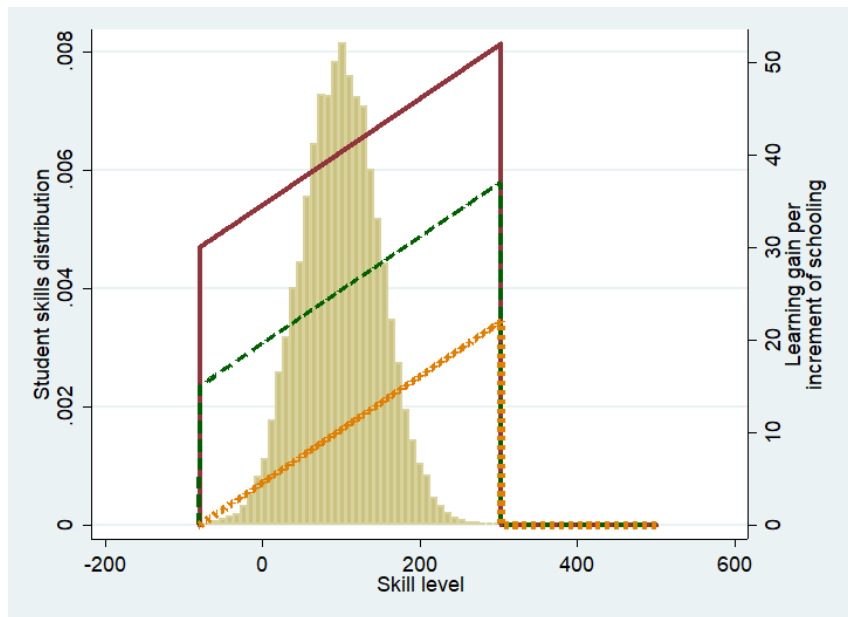
Average cumulative learning in grade 10 starts from OECD average of 500, down to average of 100 just by varying centeredness



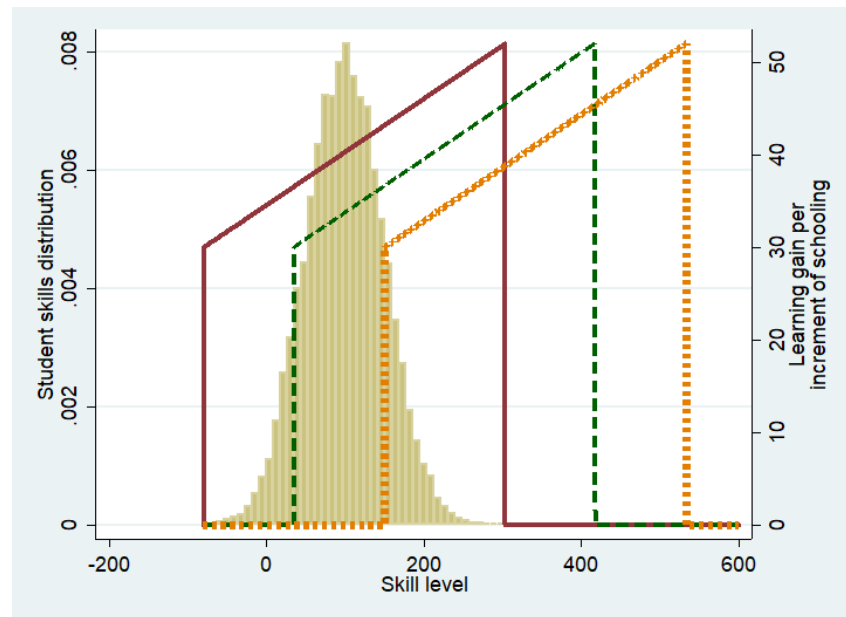
# Improving learning requires improving the right parameter

- A teacher training program aimed at improving teacher ability (height) could improve learning in Scenario A but not Scenario B
- Critical to consider appropriate parameter (height, centeredness, range, shape, pace) when attempting to improve learning

Green and yellow need improved height, e.g. through teacher training to increase ability



Green and yellow need improved centeredness, e.g. through appropriate curriculum or teaching methods



# Conclusions

- Learning outcomes are driven by the initial distribution of student skills and an instructional process that produces some level of learning for a child at each point in distribution
- Our PPF model, with parameters for height, shape, range, centeredness, and pace, can reproduce observed learning outcomes
- Application to understanding of TVA shows it is critical to not just consider one parameter of the PPF – such as “height”
  - Must consider all parameters – centeredness, range, shape, pace
  - Adjust the one(s) most critical for improving learning outcomes

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**Thank you**

# Appendix 1: Additional PPF Shapes

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# PPF<sub>3</sub>: A “pro-poor (performers)” model

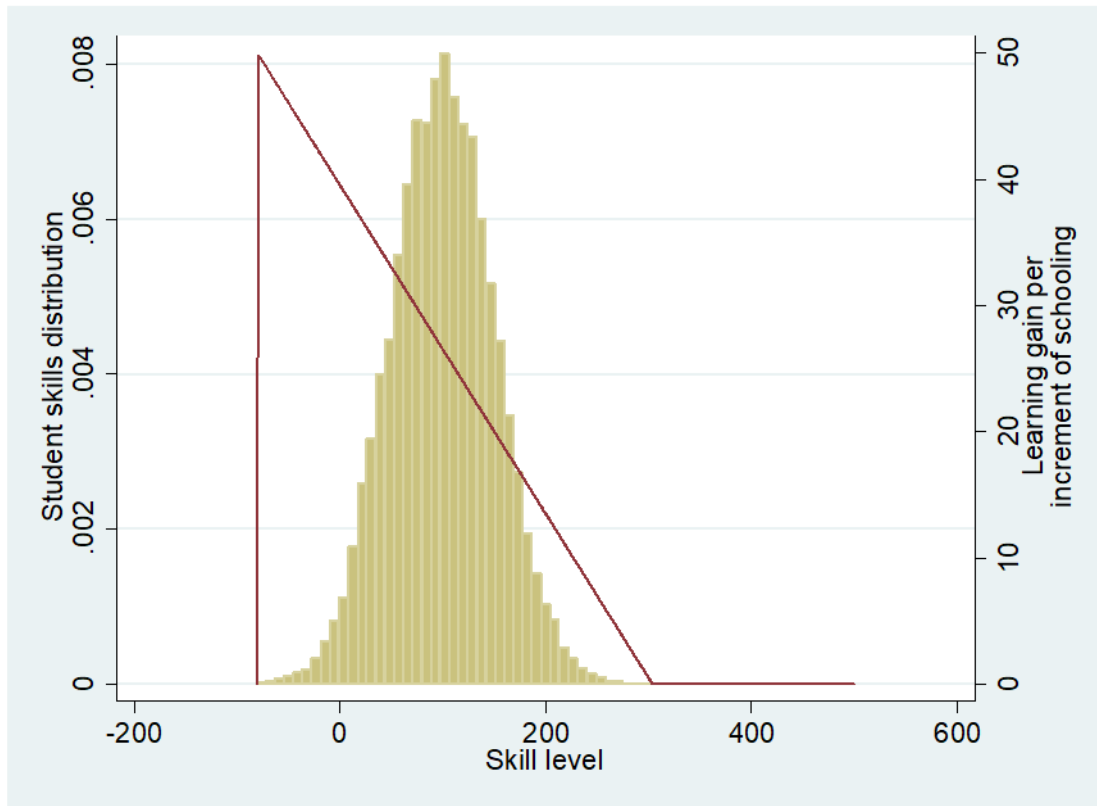
PPF<sub>3</sub>: Instruction process benefits low performers most, decreasing linearly with higher performance

- Maximum height of PPF is at left-most point, and declines linearly towards zero

$$L = \begin{cases} 0 & \text{if } s^i < a \\ (h_{max} * (b - s^i)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

# PPF<sub>3</sub>: “Pro-poor (performers)”

Figure 5: Instruction targets low performers, declines linearly



$$L = \begin{cases} 0 & \text{if } s^i < a \\ (h_{max} * (b - s^i)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

Parameters constructed to encompass full student distribution:

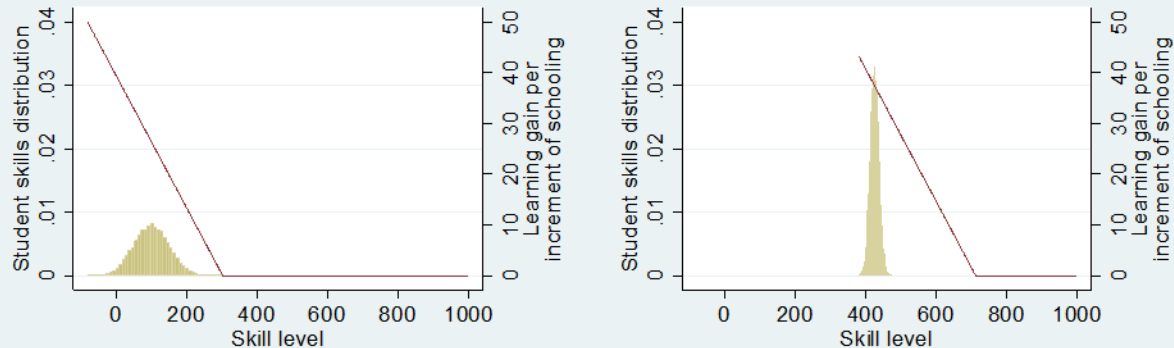
$$\begin{aligned} a &= student_{min} \\ b &= student_{max} \\ h_{max} &= 50 \end{aligned}$$

Source: Authors' simulations



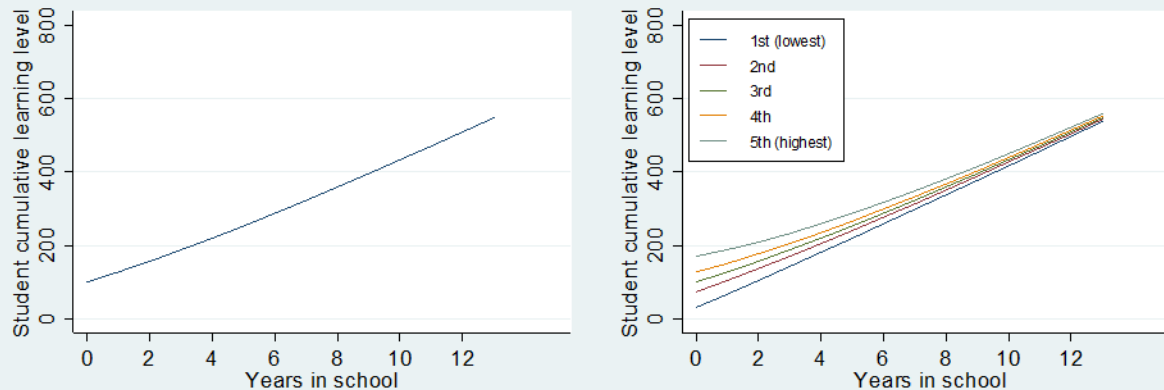
# PPF<sub>3</sub>: “Pro-poor (performers)” Learning Outcomes

Pedagogical function and student distribution at year zero and year 10



- High overall learning (average learning profile), and substantial convergence of learning (disaggregated learning profile)

Learning profiles: Average, and by quintiles



# PPF<sub>4</sub>: A pro-high-performers model

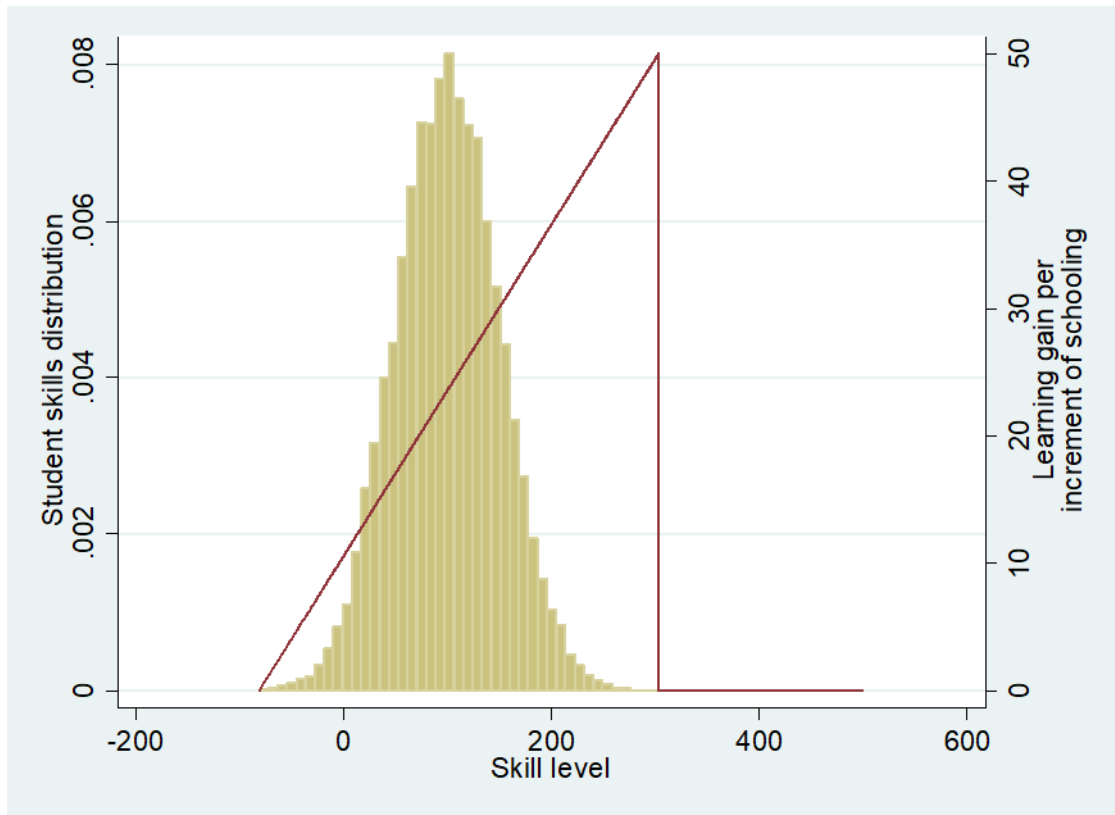
PPF<sub>4</sub>: Instruction process benefits high performers most, decreasing linearly with lower performance

- Maximum height of PPF is at right-most point, and declines linearly towards zero

$$L = \begin{cases} 0 & \text{if } s^i < a \\ (h_{max} * (s^i - a)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

# PPF<sub>4</sub>: Pro-high-performers

Figure 5: Instruction targets high performers, declines linearly



$$L = \begin{cases} 0 & \text{if } s^i < a \\ (h_{max} * (s^i - a)) / (b - a) & \text{if } a < s^i < b \\ 0 & \text{if } s^i > b \end{cases}$$

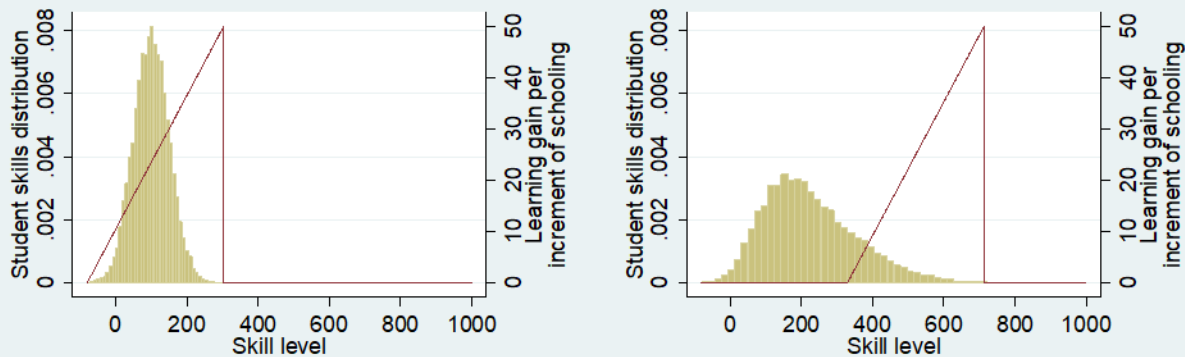
Parameters constructed to encompass full student distribution:

$$\begin{aligned} a &= student_{min} \\ b &= student_{max} \\ h_{max} &= 50 \end{aligned}$$

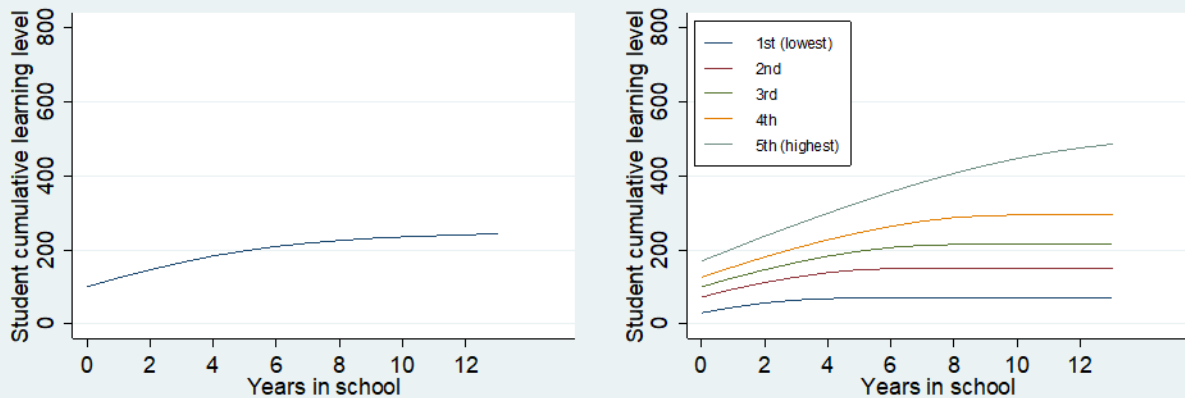
Source: Authors' simulations

# PPF<sub>4</sub>: Pro-high-performers

Pedagogical function and student distribution at year zero and year 10



Learning profiles: Average, and by quintiles



- Instruction caters to high performers
- Produces lower learning for ALL quintiles, only very top tail benefits
- By grade 6 most students are outside PPF range and have stopped learning
- Could represent over-ambitious curriculum that leaves many behind

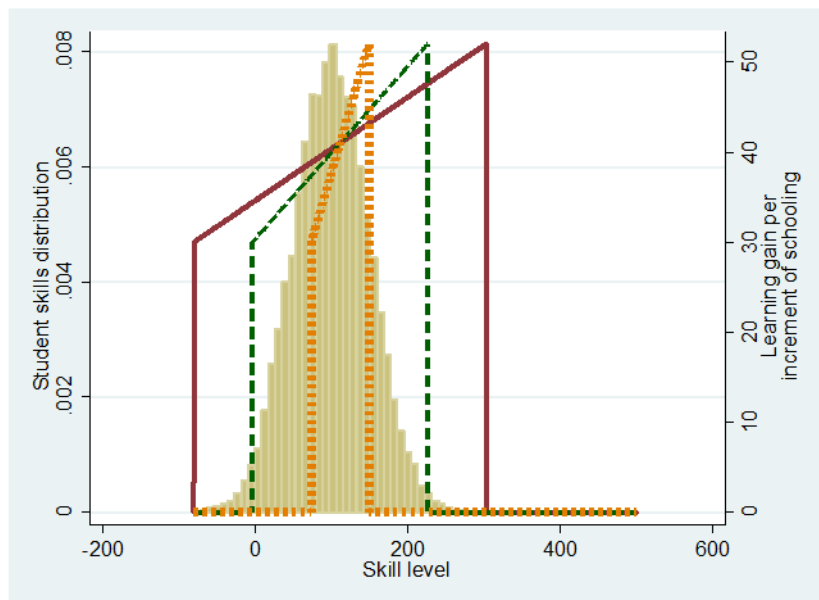
# Appendix 2: Varying range

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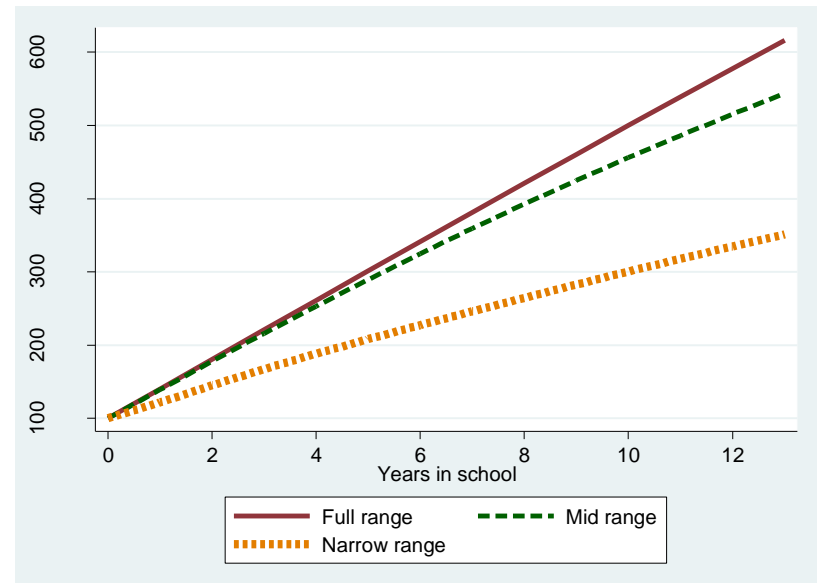
# Same max and min height, but varied range of meaningful instruction

- Some teachers are able to adapt teaching to wide range of student skills, others can only accommodate narrow range
- Simulate PPFs across 12 years of schooling to produce learning profiles
- Red replicates OECD; green and yellow have smaller range of skills they are able to instruct
- If measuring TVA, observe learning outcomes, but not underlying PPF

3 PPFs with same max and min height, but varied range – e.g. varied ability to teach students at different skill levels



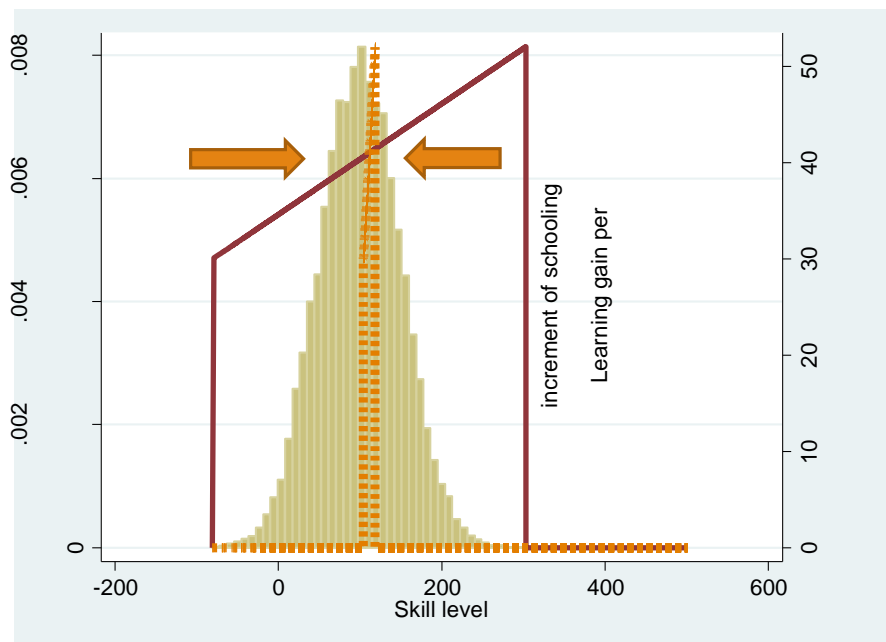
Observed TVA: Produces increasingly less learning as range shrinks



# What if we iterate across all ranges, what average learning levels are produced?

- By varying only range (other parameters constant), can produce learning outcomes in grade 10 of everything from OECD scores to almost no learning at all

Vary PPF range from full distribution to only center of distribution; other parameters constant



Gradual decline then sharp decline as more students fall outside range of PPF

